Mathematics Higher Tier, November 2008
4301/1H (Paper 1, non-calculator)

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Question 1

rounding each term to one significant figure we get $\frac{90+30}{5 \times 8} = \frac{120}{40} = \frac{12}{4} = 3$

Question 2

a) -3 → add 11 → divide by 4 =(-3 + 11) ÷ 4 = 8 ÷ 4 = 2

b) n → add 11 → divide by 4 = (n + 11) ÷ 4 = $\frac{n+11}{4}$

Question 3

Andy earns £24,000 ÷ 12 = £2,000 per month. 10% of £2000 is £200. So his new monthly salary is £2000 + £200 = £2,200

Question 4

a) and b)

	Boys	Girls	Total
Level 4	2	1	3
Level 5	1	2	3
Level 6	3	3	6
Level 7	3	5	8
total	**9**	**11**	**20**

Question 5

a) x will be the same value as 40° because alternate angles are equal

b) angles in a triangle always add up to 180° so given that x was 40° we have 40 + 75 = 115°.
y = 180 – 115 = 65°

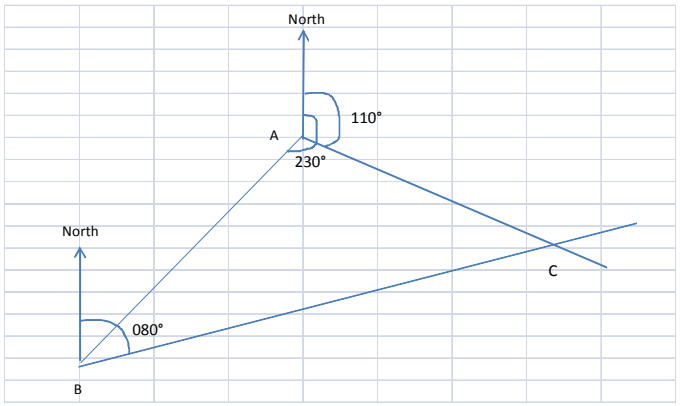
Question 6

a) angles in a triangle always add up to 180°.
We have $x + 4x + 80 = 180$
grouping terms
$5x + 80 = 180$
Subtract 80 from both sides
$5x = 100$
Divide both sides by 5
$x = 20$

b) if $x = 20°$ then we have one angle of 20°, the top angle will be 80° ($4x = 4 \times 20°$) and we already know one angle to be 80°. So we have angles of 20°, 80° and 80°. Two of the angles are the same so we have an isosceles triangle.

Question 7

a) the bearing of B from A (always measure from North) is 230°



Question 8

Common difference is 4 so we know the sequence will be of the type 4n
Write out the 4 times table
4, 8, 12, 16, …
We need to add 2 to each of these terms to get the original sequence
So we have 4n + 2

Question 9

a) multiply both sides by w
18 = 3w
Divide both sides by 3
6 = w
w = 6

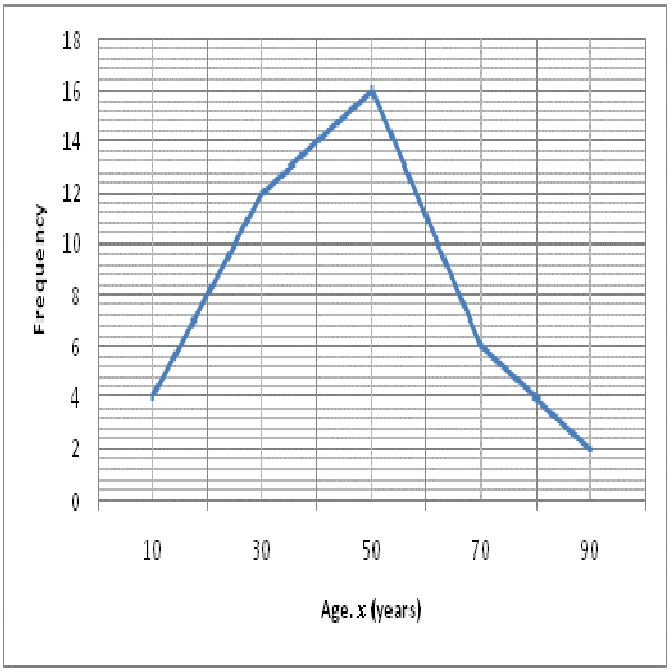
b) divide both sides by 5
x + 4 = 10 ÷ 2 = 5
subtract 4 from both sides
x = - 2

c) multiply both sides by 3
33 + y = 45
Subtract 33 from both sides
y = 12

Question 10

a) The modal class is the category that has the highest frequency. So modal class is $40 < x \leq 60$

b)



Question 11

a) area of triangle is ½ x base x height.
½ x 10 x 15 = 5 x 15 = 75 cm^2

b) area of rectangle is 10 x 15 = 150 cm^2
area of smaller triangle is ½ x base x height = ½ x (10 – 4) x 9 = ½ x 6 x 9 = 3 x 9 = 27cm^2
so shaded area is area of rectangle minus the areas of the two triangles
150 – (75 + 27) = 150 – 102 = 48 cm^2

Question 12

a) The factors of 8 are 1, 2, 4 and 8
The factors of 12 are 1, 2, 3, 4, 6 and 12
The common factors are 1, 2, 4 and the **highest** common factor is 4

b) Multiples of 8 are 8, 16, 24, 32, 40, …
Multiples of 12 are 12, 24, 36, 48….
24 is the **lowest** common multiple

Question 13

a) f(0) = (3 x 0^2) – 2 = 0 – 2 = -2

b) f(2) = (3 x 2^2) – 2 = (3 x 4) – 2 = 12 – 2 = 10

c) $3x^2 – 2 = 73$
add 2 to both sides
$3x^2 = 75$
divide both sides by 3
$x^2 = 25$
square root both sides
$x = +5$ or -5

Question 14

a) (2.1 x 10^5) + (7 x 10^4) = (21 x 10^4) + (7 x 10^4) = 28 x 10^4 = 2.8 x 10^5

b) (2.1 x 10^5) x (7 x 10^4) = 14.7 x 10^9 = 1.47 x 10^{10}

Question 15

a) "angles in a semi-circle are right angles"
angle PQR is 90^o

b) i) "angles in the same segment are equal"
angle RPQ = 27^o

ii) PRQ is a triangle and the angles in a triangle add up to 180^o. Angle PQR is 90^o and angle RPQ is 27^o so angle PRQ is $180^o - (90^o + 27^o) = 180^o - 117^o = 63^o$

c) For this question it is very important to note that the point Q is now in a different position so that the angles in part b) above no longer apply.

if QS is a diameter then angle QRS will be a right angle. Therefore angle QRO will be $90^o - 38^o = 52^o$ and OQR is an isosceles triangle (as OQ and OR are both radii so of equal length) so angle OQR = angle ORQ = 52^o

Question 16

a) from the LHS we have $(x + 1) + 2y + 11 = x + 1 + 2y + 11 = x + 2y + 12$
from the rhs we have $11 + (2x + 2y) + y = 11 + 2x + 2y + y = 2x + 3y + 11$
setting these two expressions to equal each other we have
$x + 2y + 12 = 2x + 3y + 11$
subtract x from both sides
$2y + 12 = x + 3y + 11$
subtract 2y from both sides
$12 = x + y + 11$
subtract 11 from both sides
$1 = x + y$
so $x + y = 1$

b) from the lhs we have $x + 2y + 12$
from the bottom we have $(x + 1) + 2x + y = x + 1 + 2x + y = 3x + y + 1$
setting these two expressions equal to each other we have
$x + 2y + 12 = 3x + y + 1$
subtract x from both sides
$2y + 12 = 2x + y + 1$
subtract 2y from both sides
$12 = 2x - y + 1$
subtract 1 from both sides
$11 = 2x - y$
so $2x - y = 11$

c) we can solve by elimination or by substitution

<u>elimination</u>
no need to multiply either equation by anything as already each equation has the same number of y
add the two equations (so that the y's will cancel out)
$x + y = 1$
$2x - y = 11$
$3x = 12$

divide both sides by 3
$x = 4$
now put this back into the first equation to get $4 + y = 1$
subtract 4 from both sides
$y = -3$
so $x = 4$ and $y = -3$
(put back into the second equation to check your answer ((2 x 4) – (-3) = 8 –3 = 8 + 3 = 11✓)

Substitution

we can easily rearrange the first equation to make x the subject, x = 1 – y

substitute this into the second equation

2 (1 – y) – y = 11

expand

2 – 2y – y = 11

group terms

2 – 3y = 11

add 3y to both sides

2 = 11 + 3y

subtract 11 from both sides

-9 = 3y

divide both sides by 3

-3 = y

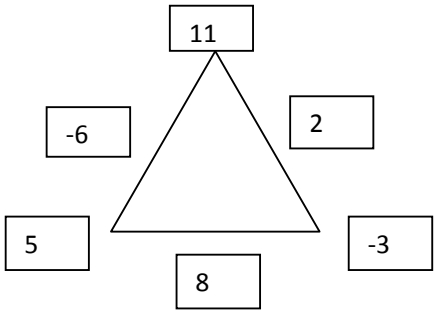
put this back into x = 1 – y

x = 1 - -3 = 1 + 3 = 4

we have x = 4 and y = -3

(put back into the second equation to check your answer ((2 x 4) – (-3) = 8 –3 = 8 + 3 = 11✓)

d)



working clockwise round from 11

$2x$ + 2y = (2 x 4) + (2 x -3) = 8 – 6 = 2

y = -3

$2x$ = 2 x 4 = 8

x + 1 = 4 + 1 = 5

2y = 2 x -3 = -6

Question 17

Anything to the power of $\frac{1}{n}$ is the same as the nth root (for example $27^{⅓} = \sqrt[3]{27} = 3$). Anything to the power of a negative number means that we first take the reciprocal of the number (for example $6^{-2} = 1/6^2 = \frac{1}{36}$)

So we have $8^{⅓} \times 2^{-5} = (\sqrt[3]{8}) \times (1/2^5) = 2 \times \frac{1}{32} = \frac{1}{16} = ¼^2 = 4^{-2}$

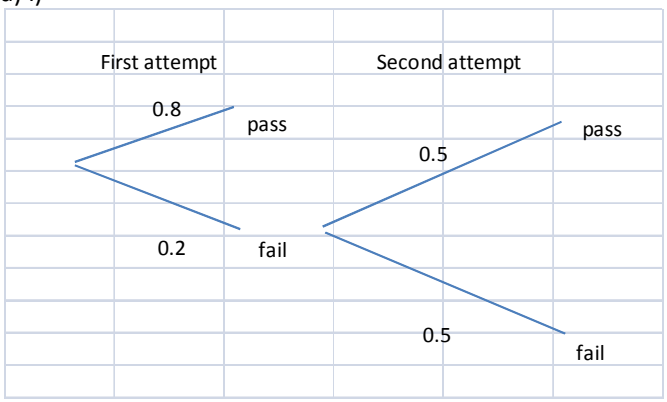
Question 18

a) This is a harder factorization question because there is a number in front of the n^2 so we have to first multiply 2 by 9 to get 18
we then must find a pair of numbers that multiply to get 18 and add to get 9
these two numbers could be +3 and +6
we rewrite the original expression splitting the middle term into +3n and +6n
$2n^2 + 3n + 6n + 9$
factorise in pairs
$n(2n + 3) + 3(2n + 3)$ we should always have the same thing in the brackets and we do $(2n + 3)$
hence the first factor is $(2n + 3)$ and the other factor is $(n + 3)$
$(2n + 3)(n + 3)$

b) there will be a number n such that $(2n + 3)(n + 3) = 299$
if we look back at $2n^2 + 9n + 9$ we can see that if $n = 10$ we would have $200 + 90 + 9 = 299$
so with $n = 10$ we have $((2 \times 10) + 3)(10 + 3) = 23 \times 13 = 299$
Both 23 and 13 are prime numbers so we have our answer.

Question 19

a) i)



ii) probability that fails both attempts is $0.2 \times 0.5 = 0.10 = 0.1$

b) the candidate could pass on first attempt with probability 0.8 or could pass on second attempt (having failed on first attempt) with probability (0.2×0.5) 0.1
overall probability of passing is $0.8 + 0.1 = 0.9$

c) we need Hassan to pass and Louise to fail or the other way around
probability that Hassan passes is 0.9, probability of Louise failing is 0.1. Probability of Hassan passing and Louise failing is $0.9 \times 0.1 = 0.09$
The probability of Louise passing and Hassan failing will also be 0.09.
Therefore the probability that one passes and the other fails is $0.09 + 0.09 = 0.18$

Question 20

a) area of grass is $(10 - x)(8 - x) = 80 + x^2 - 8x - 10x = x^2 - 18x + 80$

area of garden is 8 x 10 = 80
area of grass is $\frac{3}{5}$ of area of garden

$x^2 - 18x + 80 = \frac{3}{5}$ x 80 = 3 x 16 = 48
so $x^2 - 18x + 80 = 48$
subtract 48 from both sides
$x^2 - 18x + 32 = 0$

b) to solve we could factorise, complete the square or use the quadratic formula. I probably wouldn't use the quadratic formula as this is best used on calculator type questions.

Factorising
if we can factorise then there must be two numbers that multiply to give 32 and add to give -18.
these two numbers are -2 and -16.
so $(x - 16)(x - 2) = 0$
either $x = 16$ or $x = 2$
in the context of this question x cannot equal 16, so $x = 2$
width of path is 2m

Completing the Square
$(x - 9)^2 - (9^2) + 32 = 0$
$(x - 9)^2 - 81 + 32 = 0$
$(x - 9)^2 - 49 = 0$
add 49 to both sides
$(x - 9)^2 = 49$
square root both sides
$(x - 9) = \pm 7$
add 9 to both sides
$x = +7 + 9 = 16$ or $x = -7 + 9 = 2$
in the context of this question x cannot equal 16, so $x = 2$
width of path is 2m

Question 21

volume of a prism is area of cross section x length

area of cross section is the area of a sector of a circle = $\frac{60}{360}$ x π x r^2 = $\frac{60}{360}$ x π x 12^2 = $\frac{1}{6}$ x π x 12 x 12

= π x 2 x 12 = 24π

volume of prism = 24π x 20 = 480π cm^3

Question 22

$\frac{\sqrt{x} \times \sqrt{50}}{\sqrt{5}} = 4\sqrt{5}$

Multiply both sides by $\sqrt{5}$

$\sqrt{x} \times \sqrt{50} = 4 \times 5 = 20$

divide both sides by $\sqrt{50}$

$\sqrt{x} = 20/\sqrt{50}$

rationalise the RHS by multiplying the top and the bottom by $\sqrt{50}$

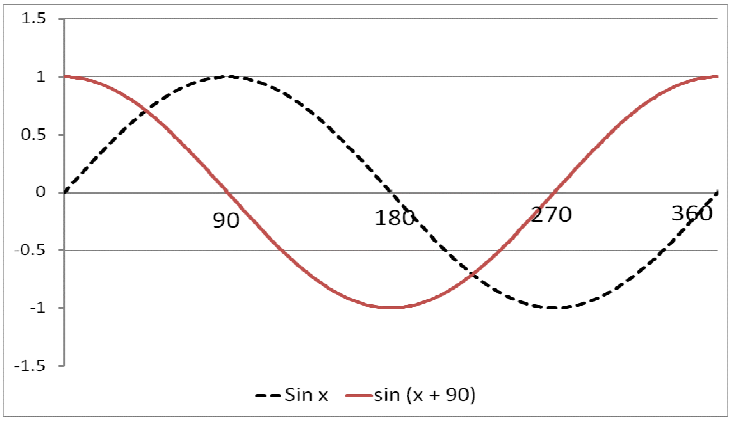
$\sqrt{x} = \frac{20 \times \sqrt{50}}{50}$

square both sides

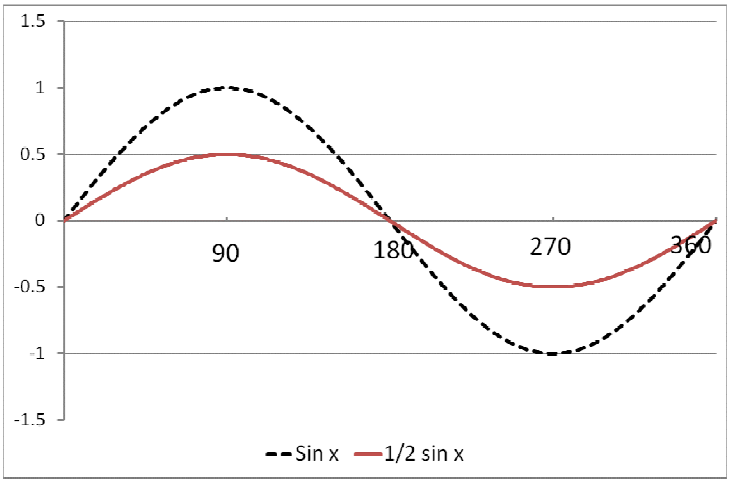
$x = \frac{20 \times \sqrt{50} \times 20 \times \sqrt{50}}{50 \times 50} = \frac{400 \times 50}{50 \times 50} = \frac{400}{50} = 8$

Question 23

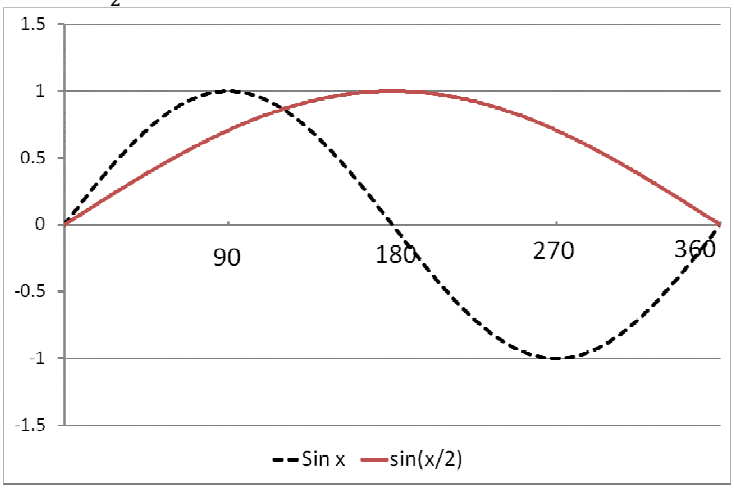
a) y = sin (x + 90) this is a translation. the change is affecting the x values and shifts the whole curve 90° in the negative x direction.



b) y = ½ sin x this is a stretch of scale ½ parallel to the y axis



c) y = sin $(\frac{x}{2})$ this is a stretch of scale factor 2 parallel to the x axis



Question 24

a) $\overrightarrow{AB}$ = $\overrightarrow{AO}$ + $\overrightarrow{OB}$ = -2a + 2b

b) $\overrightarrow{PQ}$ = $\overrightarrow{PA}$ + $\overrightarrow{AQ}$ = a + ¼ (-2a + 2b) = a - $\frac{a}{2}$ + $\frac{b}{2}$ = $\frac{a}{2}$ + $\frac{b}{2}$

c) $\overrightarrow{OM}$ = $\overrightarrow{OA}$ + $\overrightarrow{AM}$ = 2a + ½ $\overrightarrow{AB}$ = 2a + ½ (-2a + 2b) = 2a – a + b = a + b

d) A trapezium is a four sided shape that has one pair of parallel lines. OP and QM are clearly not parallel. PQ and OM are parallel. We can see this because the vector $\overrightarrow{PQ}$ can be expressed as a multiple of the vector $\overrightarrow{OM}$ ($\overrightarrow{PQ}$ = ½ $\overrightarrow{OM}$)