Question 1

Simplify fully $\frac{x^2-8x+15}{2x^2-7x-15}$

Answer:

In all of these types of questions you can usually factorise the top and the bottom and then one of the factors will cancel.

To factorise the top we need to find two numbers that multiply to give 15 and combine to give -8. These two numbers could be -3 and -5. Therefore the top factorises to give (x -3)(x -5).

To factorise the bottom, it is a bit harder as we have a number greater than 1 in front of the x^2 term. Take the coefficient of the x^2 and the units figure (2 and -15) and multiply them together. This gives -30. Now we must find two numbers that multiply to give -30 but combine to give -7 (the coefficient of the x term). These two numbers could be +3 and -10. Rewrite the quadratic equation splitting the x term into these two components:

$2x^2 - 7x - 15 = 2x^2 + 3x - 10x - 15$
factorise this into two pairs
$x(2x + 3) - 5(2x + 3)$
if this is correct then it should be the same factor for both ((2x + 3) in this case)
now factorise again
$(2x + 3)(x - 5)$

So going back to our original expression we have

$\frac{(x-3)(x-5)}{(2x+3)(x-5)}$
and as expected one of the factors (x-5) will cancel on the top and the bottom leaving us with:

$$\frac{x-3}{2x+3}$$

Question 2

Simplify fully $\frac{5x^2+14x-3}{50x^2-2}$

Answer:

We need to factorise the top and the bottom and hope that something will cancel

To factorise the top we need to first multiply 5 by -3 to get -15. We need to find two numbers that multiply to get -15 but add to get +14. These two numbers are +15 and -1.
Rewrite the expression splitting the $14x$ into $+15x$ and $-1x$
$5x^2 + 15x - 1x - 3$
factorise in pairs
$5x(x + 3) - 1(x + 3)$
we should have the same thing in both brackets, we do✓
factorise again
$(x + 3)(5x - 1)$

To factorise the bottom, we need to first factorise out the 2
$2(25x^2 - 1)$
then recognise that this is the difference of the squares
$2(5x - 1)(5x + 1)$

putting the fraction back together again in factorised form
$\frac{(x+3)\cancel{(5x-1)}}{2\cancel{(5x-1)}(5x+1)}$
we can see that $(5x - 1)$ is on the top and the bottom so can cancel each other out
we are left with

$\frac{(x+3)}{2(5x+1)}$

Question 3

Solve the equation $\frac{2}{3x-1} - \frac{3}{2x+1} = \frac{2}{5}$

Answer:

multiply through by $(3x - 1)$

$2 - \frac{3(3x-1)}{2x+1} = \frac{2(3x-1)}{5}$

multiply through by $(2x + 1)$

$2(2x + 1) - 3(3x - 1) = \frac{2(3x-1)(2x+1)}{5}$

multiply through by 5

$10(2x + 1) - 15(3x - 1) = 2(3x - 1)(2x + 1)$

expand the brackets

$10(2x + 1) - 15(3x - 1) = 2(3x - 1)(2x + 1)$

$20x + 10 - 45x + 15 = 2(6x^2 - 1 - 2x + 3x)$

group terms

$-25x + 25 = 2(6x^2 + x - 1)$

expand the brackets

$-25x + 25 = 12x^2 + 2x - 2$

add 25x to both sides

$25 = 12x^2 + 27x - 2$

subtract 25 from boths sides

$0 = 12x^2 + 27x - 27$

All the numbers are a multiple of 3 so divide by 3

$4x^2 + 9x - 9 = 0$

We can either solve this by factorising or by using the quadratic formula

Factorising

we need to find two numbers that multiply to make -36 (4 x -9) and that add to make 9

these two numbers could be -3 and +12

rewrite the equation splitting the middle term into $-3x$ and $12x$

$4x^2 - 3x + 12x - 9 = 0$

factorise in pairs

$x(4x - 3) + 3(4x - 3) = 0$

factorise again

$(4x - 3)(x + 3) = 0$

So either $4x - 3 = 0$ or $x + 3 = 0$

If $4x + 3 = 0$
$4x = 3$
$x = \frac{3}{4}$

If $x + 3 = 0$
$x = -3$

quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a = 4
b = 9
c = -9

$$x = \frac{-9 \pm \sqrt{9^2 - (4 \text{ x } 4 \text{ x} -9)}}{2 \text{ x } 4}$$

$$x = \frac{-9 \pm \sqrt{81 - -144}}{8}$$

$$x = \frac{-9 \pm \sqrt{81 + 144}}{8}$$

$$x = \frac{-9 \pm \sqrt{225}}{8}$$

$$x = \frac{-9 \pm 15}{8}$$

$x = \frac{-9+15}{8}$ or $x = \frac{-9-15}{8}$

$x = \frac{3}{4}$ or -3

Question 4

Simplify fully $\frac{2}{x+2} + \frac{x}{x^2+5x+6}$

Answer:

we can first factorise the denominator of the second fraction

$x^2 + 5x + 6 = (x + 3)(x + 2)$

we have

$$\frac{2}{x+2} + \frac{x}{(x+3)(x+2)}$$

In order to add fractions we need a common denominator
let the common denominator be $(x + 3)(x + 2)$

$$\frac{2(x+3)}{(x+3)(x+2)} + \frac{x}{(x+3)(x+2)}$$

expanding and adding the numerators

$$\frac{2x+6+x}{(x+3)(x+2)} = \frac{3x+6}{(x+3)(x+2)} = \frac{3(x+2)}{(x+3)(x+2)}$$

we can now cancel down the $(x + 2)$ to get

$$\frac{3}{(x+3)} = \frac{3}{x+3}$$

Question 5

a) simplify $\frac{x^2}{x^2 - 2x}$

b) simplify $\frac{2}{2x-1} - \frac{1}{x+1}$

Answer:

a) $\frac{x \times x}{x(x-2)} = \frac{x}{x-2}$

b) we need a common denominator, $(2x - 1)(x + 1)$

$\frac{2}{2x-1} = \frac{2(x+1)}{(2x-1)(x+1)}$

$\frac{1}{x+1} = \frac{2x-1}{(2x-1)(x+1)}$

$\frac{2(x+1)}{(2x-1)(x+1)} - \frac{2x-1}{(2x-1)(x+1)} = \frac{2(x+1)-(2x-1)}{(2x-1)(x+1)} = \frac{2x+2-2x+1}{(2x-1)(x+1)} = \frac{3}{(2x-1)(x+1)}$

Question 6

Solve the equation

$\frac{4}{x} + \frac{3}{x-2} = 1$

Answer:

go through and multiply everything by x

$4 + \frac{3x}{x-2} = x$

multiply everything by $x - 2$

$4(x - 2) + 3x = x(x - 2)$

expand the brackets

$4x - 8 + 3x = x^2 - 2x$

group terms

$7x - 8 = x^2 - 2x$

subtract $7x$ from both sides (so that we move everything on to the side where x^2 will be positive)

$-8 = x^2 - 9x$

add 8 to both sides

$0 = x^2 - 9x + 8$

$x^2 - 9x + 8 = 0$

we can factorise as we can see two numbers that multiply to make 8 and add to make -9 (-1 and -8)

$(x - 1)(x - 8) = 0$

if two things multiply to give 0 then either x -1 = 0 or $x - 8 = 0$

If $x - 1 = 0$ then $x = 1$

or $x - 8 = 0$ so $x = 8$

$x = 1$ or 8

Question 7

Simplify fully $\frac{2(x+1)^2}{10(x+1)}$

Answer:

we can cancel the 2 with the 10 to give

$\frac{(x+1)^2}{5(x+1)}$

we can cancel $(x+1)$ as well

$\frac{x+1}{5}$

Question 8

Solve $\frac{4}{x+3} + \frac{3}{2x-1} = 1$

Answer:

first multiply both sides by $(x + 3)$

$4 + \frac{3(x+3)}{2x-1} = (x + 3)$

multiply both sides by $(2x - 1)$

$4(2x - 1) + 3(x + 3) = (x + 3)(2x - 1)$

expand

$8x - 4 + 3x + 9 = 2x^2 + 6x - x - 3$

group terms

$11x + 5 = 2x^2 + 5x - 3$

subtract 11x from both sides

$5 = 2x^2 - 6x - 3$

subtract 5 from both sides

$0 = 2x^2 - 6x - 8$

rewrite

$2x^2 - 6x - 8 = 0$

divide by 2

$x^2 - 3x - 4 = 0$

factorise

$(x - 4)(x + 1) = 0$

so if two brackets multiply to give 0 then one or both of the brackets must equal 0

$x = 4$ or $x = -1$

to check: put back $x = 4$ into original equation to get $\frac{4}{7} + \frac{3}{7} = \frac{7}{7} = 1$✓

and put $x = -1$ into original equation to get $\frac{4}{2} + \frac{3}{-3} = 2 + -1 = 2 - 1 = 1$✓

Question 9

Simplify fully $\frac{6x^2+x-1}{4x^2-1}$

Answer:

We need to factorise the top and the bottom and hopefully there will be a common factor which will cancel

Factorise the top

We need to first multiply 6 by -1 to get -6. We need to find two numbers that multiply to give -6 and combine to give 1. These two numbers are +3 and -2. Rewrite the expression splitting the x term into $+3x$ and $-2x$.

$6x^2 + 3x - 2x - 1$

factorise in pairs

$3x(2x + 1) - (2x + 1)$

we should have the same thing in both brackets which we do✓

factorise again

$(2x + 1)(3x - 1)$

Factorise the bottom

this is the difference of two squares

$(2x + 1)(2x - 1)$

so we have

$$\frac{(2x+1)(3x-1)}{(2x+1)(2x-1)} = \frac{\cancel{(2x+1)}(3x-1)}{\cancel{(2x+1)}(2x-1)} = \frac{3x-1}{2x-1}$$

Question 10

Solve $\frac{x}{2}+\frac{x}{3}=\frac{5}{4}$

Answer:

in order to add fractions we need a common denominator
6 is a multiple of 2 and 3 so use 6 as the denominator

$\frac{x}{2}=\frac{3x}{6}$

$\frac{x}{3}=\frac{2x}{6}$

We have $\frac{3x}{6}+\frac{2x}{6}=\frac{5}{4}$

$\frac{5x}{6}=\frac{5}{4}$

multiply both sides by 6

$5x=\frac{30}{4}=\frac{15}{2}$

divide both sides by 5

$x=\frac{3}{2}=1.5$

Question 11

Simplify fully $1 + \frac{x^2 + x - 6}{(x+4)(x-2)}$

Answer:

In order to add fractions we need a common denominator, but before we do that we can probably simplify the fraction. We need to factorise the top and usually something will cancel with the bottom.

find two numbers that multiply to give -6 but add to give 1: the two numbers are +3 and -2

$x^2 + x - 6 = (x + 3)(x - 2)$

so we have $1 + \frac{(x+3)(x-2)}{(x+4)(x-2)}$

now we can see that the factor $(x - 2)$ will cancel with the top and the bottom to give

$1 + \frac{(x+3)}{(x+4)}$

Using a common denominator of $(x + 4)$ we can put 1 as $\frac{(x+4)}{(x+4)}$

$\frac{(x+4)}{(x+4)} + \frac{(x+3)}{(x+4)} = \frac{x+4+x+3}{x+4} = \frac{2x+7}{x+4}$

Question 12

Solve the equation $\frac{x+3}{2} - \frac{x-2}{3} = 3$

Answer:

in order to subtract (or add) fractions you must have a common denominator (number on the bottom)
the common denominator here could be 6
multiply the first fraction by 3 on the top and the same on the bottom
multiply the second fraction y 2 on the top and the same on the bottom

$\frac{3(x+3)}{6} - \frac{2(x-2)}{6} = 3$

$\frac{3(x+3)-2(x-2)}{6} = 3$
multiply both sides by 6

$3(x + 3) - 2(x - 2) = 18$
expand the brackets
$3x + 9 - 2x + 4 = 18$
group terms
$x + 13 = 18$
subtract 13 from both sides

$x = 5$

to check put this value of x back into the original equation
$\frac{5+3}{2} - \frac{5-2}{3} = \frac{8}{2} - \frac{3}{3} = 4 - 1 = 3$✓

Question 13

a) Simplify fully $\frac{(x+10)^5}{(x+10)^4}$

b) Simplify fully $\frac{x^2-25}{x^2+7x+10}$

Answer:

a) the term $(x + 10)$ cancels from the top and the bottom 4 times leaving just $x + 10$

another way of seeing this is: $\frac{\cancel{(x+10)(x+10)(x+10)(x+10)}(x+10)}{\cancel{(x+10)(x+10)(x+10)(x+10)}} = x + 10$

b) for this type of question we need to factorise the top and the bottom and then usually one of the factors will cancel

$x^2 - 25$ this is the difference of the square and factorises to give $(x - 5)(x + 5)$

$x^2 + 7x + 10$ we need to find two numbers that multiply to give +10 and add to give +7. The two numbers are 2 and 5. So we have $(x + 2)(x + 5)$

so our fraction becomes $\frac{(x-5)(x+5)}{(x+2)(x+5)}$

$(x + 5)$ cancels to give $\frac{(x-5)\cancel{(x+5)}}{(x+2)\cancel{(x+5)}} = \frac{(x-5)}{(x+2)}$

Question 14

Solve $\frac{x+1}{2} - \frac{x-3}{5} = 2$

Answer:

multiply both sides by 2

$(x + 1) - \frac{2(x-3)}{5} = 4$

multiply both sides by 5

$5(x + 1) - 2(x - 3) = 20$

expand the brackets

$5x + 5 - 2x + 6 = 20$

group terms

$3x + 11 = 20$

subtract 11 from both sides

$3x = 9$

divide both sides by 3

$x = 3$

remember to check your answer: $\frac{3+1}{2} + \frac{3-3}{5} = 2$, $2 + 0 = 2$✓

Question 15

Solve $\frac{2x-1}{4} + \frac{x+2}{3} = 2$

Answer:

to get rid of the fractions go through and multiply by 4 (and then by the 3)
multiply by 4
$(2x - 1) + \frac{4(x+2)}{3} = 8$
multiply by 3
$3(2x - 1) + 4(x + 2) = 24$
expand brackets
$6x - 3 + 4x + 8 = 24$
group terms
$10x + 5 = 24$
subtract 5 from both sides
$10x = 19$
divide both sides by 10
$x = 1.9$

Question 16

Show that the following identity hold for all values of x

$$\frac{x-3}{x} - \frac{x-2}{x+2} \equiv \frac{x-6}{x(x+2)}$$

Answer:

The ≡ sign means that we have an identity rather than an equation. An identity means that the LHS equals the RHS for all values of x. Whereas an equation means that there is only one (or a finite number of solutions that x could be). For example $6x + 3x \equiv 9x$ is an identity because it works for all x but $6x + 3x = 9$ is an equation because it only works for one value of x.

In order to add or subtract fractions you need a common denominator. Let the common denominator be $x(x + 2)$ (which is just the product of the two denominators)

$\frac{x-3}{x} = \frac{(x-3)(x+2)}{x(x+2)}$ and $\frac{x-2}{x+2} = \frac{x(x-2)}{x(x+2)}$

Now we can subtract the two fractions to get

$\frac{(x-3)(x+2) - x(x-2)}{x(x+2)}$, working on just the numerator we have

$(x-3)(x+2) - x(x-2) = x^2 - 3x + 2x - 6 - x^2 + 2x$

grouping terms

$x - 6$

putting back together again with the denominator we have

$$\frac{x-6}{x(x+2)}$$

Question 17

Simplify fully $\frac{x}{6}+\frac{3x}{4}$

Answer:

We need a common denominator (number on the bottom)
Convert both so that they have a common denominator of 12

$\frac{x}{6}=\frac{2x}{12}$

$\frac{3x}{4}=\frac{9x}{12}$

now we can add them to get
$\frac{2x}{12}+\frac{9x}{12}=\frac{11x}{12}$

Question 18

Solve the equation

$\frac{x+4}{5} + \frac{x-2}{3} = 4$

Answer:

go through and multiply both sides by 5 to get

$(x + 4) + \frac{5(x - 2)}{3} = 20$

multiply both sides by 3

$3(x + 4) + 5(x - 2) = 60$

expand

$3x + 12 + 5x - 10 = 60$

group terms

$8x + 2 = 60$

subtract 2 from both sides

$8x = 58$

divide both sides by 8

$x = 7.25$

Question 19

Simplify fully $\frac{x^2-9}{x^2+3x}$

Answer:

for these type of questions we need to factorise both the top and the bottom and then something is bound to cancel down

$x^2 - 9 = (x + 3)(x - 3)$ by difference of the square

$x^2 + 3x = x(x + 3)$ as x goes into both terms

so we have

$\frac{(x+3)(x-3)}{x(x+3)} = \frac{x-3}{x}$

Question 20

Simplify fully

$$\frac{12x^2-36x+15}{12x^2-3}$$

Answer:

for this type of question you usually need to factorise the top and the bottom and they are bound to have one factor in common which you can cancel

working on the numerator:
to factorise $12x^2 - 36x + 15$ we can first see that 3 goes into all terms so we can first take that out
$3(4x^2 - 12x + 5)$
Then work on factorising $4x^2 - 12x + 5$
As we have a number in front of the x^2, it makes the factorisation a bit harder. We must first multiply 4 and 5 together to get 20 then we need to find two numbers that multiply together to give 20 and add together to give -12. This would be -2 and -10.
Rewrite $4x^2 - 12x + 5$ as $4x^2 - 2x - 10x + 5$ (this is just splitting $-12x$ into the two components $-2x$ and $-10x$)
Now factorise in pairs
$2x(2x - 1) - 5(2x - 1)$
We should have the same in both brackets (which we do)
As $(2x - 1)$ goes into both terms we can factorise that out to get
$(2x - 1)(2x - 5)$
Remember we had already factorised the 3 out so the numerator should be
$3(2x - 1)(2x - 5)$

Working on the denominator:
First we can factorise 3 out to give $3(4x^2 - 1)$
$4x^2 - 1$ is the difference of two squares so can be expressed as $(2x - 1)(2x + 1)$
We have
$3(2x - 1)(2x + 1)$

Putting the numerator and denominator back together
$3(2x - 1)(2x - 5)$ =
$3(2x - 1)(2x + 1)$

~~$3(2x - 1)$~~$(2x - 5)$
~~$3(2x - 1)$~~$(2x + 1)$

We can see that the 3s will cancel out and so will $(2x - 1)$ so we are left with
$$\frac{2x - 5}{2x + 1}$$

Question 21

Simplify fully $\frac{x^2+6x}{x^2-36}$

Answer:

factorise the top and the bottom

$x^2 + 6x = x(x + 6)$

$x^2 - 36 = (x + 6)(x - 6)$ difference of the square

so we have

$$\frac{x(x+6)}{(x+6)(x-6)} = \frac{x}{(x-6)}$$

Question 22

Solve $\frac{2}{x} - \frac{1}{x+1} = 1$

Answer:

first multiply by x

$2 - \frac{x}{x+1} = x$

now multiply by $(x + 1)$

$2(x + 1) - x = x(x + 1)$

expand the brackets

$2x + 2 - x = x^2 + x$

group terms

$x + 2 = x^2 + x$

subtract x from both sides

$2 = x^2$

$x^2 = 2$

square root both sides

$x = \pm 1.414$

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