Question 1

Simplify the following

a) $\frac{x^6}{x^2}$

b) $3y^2z^4 \times 2y^5z$

c) $(2p^3r^2)^3$

Answer:

a) when we divide numbers (or letters) with powers we subtract the powers.
$x^6 \div x^2 = x^{6-2} = x^4$

b) we need to group the terms,
3 x 2 = 6
when we multiply numbers (or letters) with powers we add the powers
$y^2 \times y^5 = y^{2+5} = y^7$
$z^4 \times z = z^4 \times z^1 = z^{4+1} = z^5$

putting back together again:
$6 \times y^7 \times z^5 = 6y^7z^5$

c)Notice that $2p^3r^2$ is all in brackets, this means that it is all to the power of 3

so we have $2^3 \times p^{3x3} \times r^{2x3} = 8p^9r^6$

alternatively we could have written:
$(2p^3r^2) \times (2p^3r^2) \times (2p^3r^2) = 2 \times 2 \times 2 \times p^3 \times p^3 \times p^3 \times r^2 \times r^2 \times r^2 = 8p^9r^6$

Question 2

a) evaluate $(9^{1/2})^4$

b) express 5^{20} as a power of 25

c) express $\sqrt{8}$ as a power of 2

Answer:

a) when we raise powers to further powers we multiply the powers together

½ x 4 = 2

$(9^{½})^4 = 9^2 = 81$

b) $5^{20} = (25^{½})^{20} = 25^{10}$

c) $\sqrt{8} = 8^{½} = (2^3)^{½} = 2^{3 \times ½} = 2^{1.5}$

Question 3

Evaluate $5^{-2} \times 100^{0.5}$
Giving your answer in its simplest form

Answer:

$5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

$100^{0.5} = \sqrt{100} = 10$

$5^{-2} \times 100^{0.5} = \frac{1}{25} \times 10 = \frac{10}{25} = \frac{2}{5}$

Question 4

What is the reciprocal of 0.8

Answer:

The reciprocal of a number is 1 over that number
For example the reciprocal of 5 is $\frac{1}{5}$
the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$

the reciprocal of 0.8 is $\frac{1}{0.8}$

we can't leave this as it is because we don't want a mix of decimals in a fraction
multiply the top and bottom by 10

$\frac{1}{0.8} = \frac{10}{8} = \frac{5}{4}$ (= 1.25)

Question 5

a) Write $\frac{1}{16}$ as a power of 2

b) Write 2 as a power of 8

Answer:

a) $2^4 = 16$

$\frac{1}{16} = \frac{1}{2^4} = 2^{-4}$

b) $2^3 = 8$

so if we cube root ($\sqrt[3]{\ }$) both sides

$2 = \sqrt[3]{8} = 8^{1/3}$

Question 6

Show that $27^{-2/3} = \frac{1}{9}$

Answer:

We can split the- $\frac{2}{3}$ into $\frac{1}{3}$ x -2

$27^{-2/3} = (27^{1/3})^{-2} = (\sqrt[3]{27})^{-2} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

Question 7

Simplify

a) $m^3 \times m^4$

b) $p^7 \div p^3$

c) $4x^2y^3 \times 3xy^2$

Answer:

a) when you multiply with powers you add them

$m^{3+4} = m^7$

b) when you divide with powers you subtract them

$p^{7-3} = p^4$

c) $4 \times 3 \times x^{2+1} \times y^{3+2} = 12x^3y^5$

Question 8

Evaluate

a) $64^{-2/3}$

b) $16^{3/4}$

c) $(\frac{125}{27})^{-1/3}$

Answer:

a) We can split the- $\frac{2}{3}$ into $\frac{1}{3}$ x -2

$64^{-2/3} = (64^{1/3})^{-2} = (\sqrt[3]{64})^{-2} = 4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

b) we can split the $\frac{3}{4}$ into $\frac{1}{4}$ x 3

$16^{3/4} = (16^{1/4})^3 = (\sqrt[4]{16})^3 = 2^3 = 8$

c) the negative power sends the number to the bottom (reciprocal)

$(\frac{125}{27})^{-1/3} = (\frac{27}{125})^{1/3}$

The power of $\frac{1}{3}$ means the cube root of

$(\frac{27}{125})^{1/3} = \sqrt[3]{\frac{27}{125}}$

We can cube root the top and the bottom separately

$\sqrt[3]{\frac{27}{125}} = \frac{3}{5}$

Question 9

a) Simplify $3c^5d \times c^2d^4$

b) Simplify $(2x^3y)^4$

c) Simplify fully $\frac{2x-6}{x^2-3x}$

Answer:

a) $3c^{5+2}d^{1+4} = 3c^7d^5$

b) $2^4 x^{3\times4}y^4 = 16x^{12}y^4$

c) we need to factorise the top and the bottom and then something will cancel

$2x - 6 = 2(x - 3)$

$x^2 - 3x = x(x - 3)$

so we have:

$\frac{2(x-3)}{x(x-3)} = \frac{2\cancel{(x-3)}}{x\cancel{(x-3)}} = \frac{2}{x}$

Question 10

a) Show that $9^{3/2} = 27$

b) Hence, or otherwise, solve the equation $9^x = 27^4$

Answer:

a) we can split $\frac{3}{2}$ into $\frac{1}{2}$ x 3

$9^{3/2} = (9^{1/2})^3$

The power of ½ is the same as square root of

$(9^{1/2})^3 = (\sqrt{9})^3 = 3^3 = 27$

b) $9^x = 27^4$ now we know from a) that $27 = 9^{3/2}$ so replacing 27 with $9^{3/2}$

we have $9^x = (9^{3/2})^4$

when you have a power and you raise it to another power then you multiply the two powers together

$\frac{3}{2}$ x 4 = 6

$(9^{3/2})^4 = 9^6$

so $x = 6$

Question 11

Simplify

a) $t^6 \times t^2$

b) $\frac{m^8}{m^3}$

c) $(2x)^3$

d) $3a^2h \times 4a^5h^4$

Answer:

a) when you multiply with powers you add the powers

$t^{6+2} = t^8$

b) when you divide with powers you subtract the powers

$m^{8-3} = m^5$

c) $(2x)^3 = 2^3 \times x^3 = 8x^3$

d) $3 \times 4 \times a^{2+5} \times h^{1+4} = 12a^7h^5$

Question 12

a) Simplify $4x^3y^5 \times 3x^2y$

b) Simplify $(27q^6)^{2/3}$

Answer:

a) $4 \times 3 \times x^{3+2} \times y^{5+1} = 12x^5y^6$

c) Everything inside the brackets is raised to the power of $\frac{2}{3}$
also if you have a power raised to another power then you multiply those powers
$6 \times \frac{2}{3} = 4$

$27^{2/3} \times q^4$

we can split $\frac{2}{3}$ into $\frac{1}{3} \times 2$

$27^{2/3} = (27^{1/3})^2 = (\sqrt[3]{27})^2 = 3^2 = 9$

So we have

$9 \times q^4 = 9q^4$

Question 13

Simplify fully $5x^4y^2 \times 3x^3y^7$

Answer:

5 x 3 = 15
$x^4 \times x^3 = x^{4+3} = x^7$
$y^2 \times y^7 = y^{2+7} = y^9$

$15x^7y^9$

Question 14

Evaluate $(\frac{1}{2})^{-4}$

Answer:

A negative power turns the fraction upside down

$(\frac{1}{2})^{-4} = (\frac{2}{1})^4 = 2^4 = 16$

Question 15

a) Simplify $p^5 \times p^4$

b) Simplify $q^5 \div q^2$

c) Simplify $12tu^6 \div 6tu^5$

d) Simplify $(9w^2y^6)^{1/2}$

e) for $a > 1$ put the following in order of size, smallest first:

a^0, a^2, a, a^{-2}, $a^{1/2}$

Answer:

a) when you multiply with powers you add the powers

$p^{5+4} = p^9$

b) when you divide with powers you subtract the powers

$q^{5-2} = q^3$

c) $12 \div 6 = 2$

$t \div t = 1$

$u^6 \div u^5 = u^1 = u$

so we have

$2 \times 1 \times u$

$2u$

d) everything inside the brackets is raised to the power of ½

$9^{1/2} \times w^{2 \times ½} \times y^{6 \times ½}$

$3 \times w^1 \times y^3$

$3wy^3$

e) $a^0 = 1$

$a^{-2} = \frac{1}{a^2}$

$a^{1/2} = \sqrt{a}$

this might be easier to see with a number in place of the a, let a be 4 for example

then we have 1, 16, 4, $\frac{1}{16}$, 2

so we can now put them in order
$\frac{1}{a^2}$, 1, $\sqrt{a}$, a, a^2

And back in the original form we have

a^{-2}, a^0, $a^{1/2}$, a, a^2

Question 16

i) Simplify $c^5 \times c^6$

ii) Simplify $e^{12} \div e^4$

Answer

i) $c^{5+6} = c^{11}$

ii) $e^{12-4} = e^8$

Question 17

a) Write as a single power of x
i) $x^6 \times x^{-2}$
ii) $x^8 \div x^{-4}$

b) simplify $(3x^2y)^3$

Answer

i) $x^{6 + -2} = x^4$

ii) $x^{8 - -4} = x^{12}$

b) everything inside the brackets is to the power of 3

$3^3 \times x^{2 \times 3} \times y^3 = 27x^6y^3$

Alternatively we could have written
$(3x^2y) \times (3x^2y) \times (3x^2y) = 3 \times 3 \times 3 \times x^2 \times x^2 \times x^2 \times y \times y \times y = 27x^{2+2+2}y^3 = 27x^6y^3$

Question 18

Evaluate

a) 5^0
b) 2^{-1}

Answer

a) anything to the power of 0 is always 1 so $5^0 = 1$

b) negative powers turn the number upside down (gives the reciprocal)
$2^{-1} = \frac{1}{2}$

Question 19

Show that $8^{1/3} \times 2^{-5} = 4^{-2}$

Answer

Anything to the power of $\frac{1}{n}$ is the same as the nth root (for example $27^{⅓} = \sqrt[3]{27} = 3$). Anything to the power of a negative number means that we first take the reciprocal of the number (for example $6^{-2} = \frac{1}{6^2} = \frac{1}{36}$)

So we have $8^{⅓} \times 2^{-5} = (\sqrt[3]{8}) \times (\frac{1}{2^5}) = 2 \times \frac{1}{32} = \frac{1}{16} = \frac{1}{4^2} = 4^{-2}$

Question 20

a) write $3^8 \times 3^6$ as power of 3

b) write $\frac{7^5}{7^2}$ as a power of 7

c) if $\frac{5^n + 5^3}{5^7} = 5^2$ then find the value of n

Answer

a) $3^{8+6} = 3^{14}$

b) $7^{5-2} = 7^3$

c) $\frac{5^{n+3}}{5^7} = 5^{n+3-7} = 5^2$

we have 5 to the power of something on both sides of the equation so the powers must be equal

$n + 3 - 7 = 2$

$n - 4 = 2$

add 4 to both sides

$n = 6$

Question 21

a) Simplify $(5a^4b)^3$

b) Evaluate $81^{0.5} \times 6^{-2}$

Give your answers in their simplest form

Answer

a) $5^3 \times a^{4 \times 3} \times b^3 = 125a^{12}b^3$

alternatively
$5a^4b \times 5a^4b \times 5a^4b = 5 \times 5 \times 5 \times a^4 \times a^4 \times a^4 \times b \times b \times b = 125a^{12}b^3$

b)
anything to the power of 0.5 means the square root of

a negative power has the effect of flipping the number (finding the reciprocal)

$81^{0.5} = \sqrt{81} = 9$

$6^{-2} = \frac{1}{6^2} = \frac{1}{36}$

so we have

$9 \times \frac{1}{36} = \frac{1}{4}$

Question 22

Simplify
a) p x p x p x p

b) 2c x 3d

Answer

a) p^4

b) 2 x 3 x c x d = 6cd

Question 23

Simplify $(2ab^2c^3)^3$

Answer

Everything inside the brackets is cubed (to the power of 3)
$2^3 \times a^3 \times b^{2 \times 3} \times c^{3x3} = 8a^3b^6c^9$

Alternatively
$(2ab^2c^3) \times (2ab^2c^3) \times (2ab^2c^3) = 2 \times 2 \times 2 \times a \times a \times a \times b^2 \times b^2 \times b^2 \times c^3 \times c^3 \times c^3 = 8a^3b^6c^9$

Question 24

Evaluate

a) 6^0

b) $64^{1/2}$

c) $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$

Answer

i) anything to the power of 0 is always 1
$6^0 = 1$

ii) anything to the power of $\frac{1}{n}$ means the nth root of it ($\sqrt[n]{64}$)
so anything to the power of ½ means the square root of it
$\sqrt{64} = 8$

iii) a negative power means the reciprocal of the positive power (we put the power to the bottom or flip the fraction over)
eg $5^{-2} = \frac{1}{5^2}$
$\left(\frac{27}{8}\right)^{-\frac{2}{3}} = \left(\frac{8}{27}\right)^{\frac{2}{3}} = \left(\left(\frac{8}{27}\right)^{\frac{1}{3}}\right)^2$

Anything to the power of $\frac{1}{3}$ means the 3rd root of it (cube root)

$\left(\sqrt[3]{\frac{8}{27}}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

Question 25

Sue says for any number a; a^2 is always less than a^3
For example, when a = 3, $3^2 < 3^3$ as $9 < 27$
Find an example to show that Sue is wrong

Answer

Sue is right for all numbers bigger than 1, but she would be wrong for any number less than 1 (this includes negative numbers)

For example -3:
$(-3)^2 = 9$
$(-3)^3 = -27$
And $9 > -27$

Another example ½:
$(\frac{1}{2})^2 = \frac{1}{4}$
$(\frac{1}{2})^3 = \frac{1}{8}$

And $\frac{1}{4} > \frac{1}{8}$

Question 26

Given that $p = 5^m$ and $q = 5^n$
Write each of these as a single power of 5
i) $\frac{p}{q}$
ii) q^2

Answer

a) $p \div q = 5^{m-n}$

b) $q^2 = (5^n)^2 = 5^{2n}$

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