Question 1

Solve the following simultaneous equations

$2y = x + 6$
$y = 2x - 3$

Answer:

label both equations eqn 1 and eqn 2
we can solve simultaneous equations by elimination or by substitution

elimination method
multiply eqn 1 by 2 so that we have $2x$ in both equations (and relabel as eqn 3)

$4y = 2x + 12$ eqn 3
$y = 2x - 3$ eqn 2

now that we have $2x$ in both equations we can eliminate the x terms by subtracting eqn 2 from eqn 3

$4y - y = (2x + 12) - (2x - 3)$
$3y = 12 - - 3 = 12 + 3 = 15$
divide both sides by 3
$y = 5$

put back into eqn 1 to get x
$(2 \times 5) = x + 6$
$10 = x + 6$
subtract 6 from both sides
$x = 4$

$x = 4$, $y = 5$

to check, put both back into eqn 2: $5 = (2 \times 4) - 3 = 8 - 3 = 5$✓

substitution method

from eqn 2 we have y = 2x – 3
substitute this value for y into eqn 1
2 x (2x – 3) = x + 6
expand the brackets
4x – 6 = x + 6
subtract x from both sides
3x - 6 = 6
add 6 to both sides
3x = 12
divide both sides by 3
x = 4

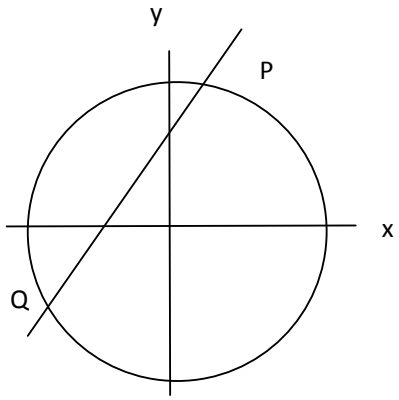
put back into eqn 2 to get y
y = (2 x 4) – 3 = 8 – 3 = 5

x = 4, y = 5

to check, put both back into eqn 1: (2 x 5) = 4 + 6 = 10✓

Question 2

The circle $x^2 + y^2 = 26$ and the line $y = x + 4$ meet at two points P and Q



a) show that the x co-ordinates of P and Q satisfy the equation $x^2 + 4x - 5 = 0$

b) hence find the co-ordinates of P and Q

Answer:

a)This is effectively two simultaneous equations
$x^2 + y^2 = 26$ (eq^n 1)
and $y = x + 4$ (eq^n 2)
we can only solve by substitution. elimination won't work here as we have x^2 and y^2 in one equation and x and y in the other)
substitute the value for y (from eq^n 2) into eq^n 1
$x^2 + (x + 4)^2 = 26$
$x^2 + (x + 4)(x + 4) = 26$
expand the brackets
$x^2 + (x + 4)(x + 4) = 26$

$x^2 + x^2 + 16 + 4x + 4x = 26$
grouping terms
$2x^2 + 8x + 16 = 26$
subtract 26 from both sides
$2x^2 + 8x - 10 = 0$
divide both sides by 2
$x^2 + 4x - 5 = 0$

b)we have a quadratic to solve, we can factorise, complete the square or use the quadratic formula.

Factorise

It will factorise if we can find two numbers that multiply to make -5 and add to make +4. The two numbers are +5 and -1

so we have $(x + 5)(x - 1) = 0$

If two things multiply to give 0 then one of them must be equal to 0

so $x + 5 = 0$ or $x - 1 = 0$.

either $x = -5$ or $x = 1$

Completing the square

$(x + 2)^2 - 2^2 - 5 = 0$

$(x + 2)^2 - 4 - 5 = 0$

$(x + 2)^2 - 9 = 0$

add 9 to both sides

$(x + 2)^2 = 9$

square root both sides

$x + 2 = \pm 3$

subtract 2 from both sides

$x = 1$ or $x = -5$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a = 1, b = 4 and c = -5

$$x = \frac{-4 \pm \sqrt{4^2 - (4\text{x}1\text{x} - 5)}}{2\text{x}1}$$

$$x = \frac{-4 \pm \sqrt{16 - -20}}{2}$$

$$x = \frac{-4 \pm \sqrt{36}}{2}$$

$$x = \frac{-4 \pm 6}{2}$$

$x = \frac{2}{2}$ or $x = \frac{-10}{2}$

$x = 1$ or x = -5

when $x = 1$, from y = x + 4, y = 1 + 4 = 5

when x = -5, y = -5 + 4 = -1

our two co-ordinates are (1, 5) and (-5, -1)

From the diagram P must be (1,5) and Q must be (-5, -1)

Question 3

Solve the following simultaneous equations

$3x - 2y = 9$
$x + 4y = 10$

Answer:

we can solve this by elimination or by substitution

Elimination
multiply the second equation by 3 so that we have the same number for x in both equations
$3x + 12y = 30$
$3x - 2y = 9$ (first equation unchanged)
we have the same number for x so can eliminate the x by subtracting the two equations
$12y - - 2y = 30 - 9$
$14y = 21$
divide both sides by 14
$y = \frac{21}{14} = \frac{3}{2} = 1.5$

now substitute this value back into the first equation

$3x - (2 \times 1.5) = 9$
$3x - 3 = 9$
add 3 to both sides
$3x = 12$
divide both sides by 3
$x = 4$

we have **$x = 4$ and $y = 1.5$**
put both back into the second equation to check: $4 + (4 \times 1.5) = 4 + 6 = 10$✓

by substitution

first rearrange one of the equations so that either x or y is the subject

I shall rearrange the second equation to get

$x = 10 - 4y$

substitute this value for x into the first equation

$3(10 - 4y) - 2y = 9$

$30 - 12y - 2y = 9$

$30 - 14y = 9$

add 14 y to both sides

$30 = 9 + 14y$

subtract 9 from both sides

$21 = 14y$

$14y = 21$

divide both sides by 14

$y = \frac{21}{14} = \frac{3}{2} = 1.5$

now substitute this value back into the first equation (rearranged)

$x = 10 - (4 \times 1.5) = 10 - 6 = 4$

we have $\boldsymbol{x}$ **= 4 and y = 1.5**

put both back into the second equation to check: $4 + (4 \times 1.5) = 4 + 6 = 10$✓

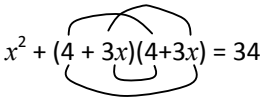
Question 4

Solve the following simultaneous equations

$y - 3x = 4$
$x^2 + y^2 = 34$

Answer:

We cannot use the elimination method here as we have x^2 in one equation and x in the other (and the same for y and y^2). We have to use the substitution method
rearrange the first equation to make y the subject
add $3x$ to both sides
$y = 4 + 3x$
now substitute this value for y into the second equation
$x^2 + (4 + 3x)^2 = 34$



$x^2 + 16 + 12x + 12x + 9x^2 = 34$
$10x^2 + 24x + 16 = 34$
subtract 34 from both sides
$10x^2 + 24x - 18 = 0$
divide all terms by 2
$5x^2 + 12x - 9 = 0$
Factorise: find two numbers that multiply to make -45 (5 x -9) and combine to make 12
These two numbers are +15 and -3
Rewrite with the $12x$ term split into $15x$ and $-3x$
$5x^2 + 15x - 3x - 9 = 0$
factorise in pairs
$5x(x + 3) - 3(x + 3) = 0$
we should have the same thing in both brackets – we do✓
factorise again
$(x + 3)(5x - 3) = 0$
If two brackets multiply to give 0 then either one or the other of the brackets must equal 0
If $x + 3 = 0$ then $x = -3$
or if $5x - 3 = 0$, then $5x = 3$, $x = \frac{3}{5}$

now substitute both values for x back into y = 4 + 3x to find the corresponding y values

when x = -3
y = 4 + (3 x -3) = 4 – 9 = -5

x = -3, y = -5

when $x = \frac{3}{5}$
$y = 4 + (3 \times \frac{3}{5}) = 4 + \frac{9}{5} = 4 + 1\frac{4}{5} = 5\frac{4}{5}$
$x = \frac{3}{5}$, $y = 5\frac{4}{5}$

to check put the values for x and y back into the second equation
$(-3)^2 + (-5)^2 = 9 + 25 = 34$✓ and
$(\frac{3}{5})^2 + (\frac{29}{5})^2 = \frac{9}{25} + \frac{841}{25} = \frac{850}{25} = 34$✓

Question 5

Solve the following simultaneous equations

5x + 4y = 3
x – 2y = 2

Answer:

we can solve simultaneous equations by using the elimination method or the substitution method

elimination method
Multiply one or both equations by a number (never the same number for both) so that we have the same number for x or the same number for y
Here I will multiply the second equation by 5 so that I can eliminate the x terms

5x + 4y = 3 (unchanged)
5x – 10y = 10 (everything multiplied by 5)

we have 5x in both equations
subtract the second equation from the first equation to make the x terms disappear

4y - - 10y = 3 – 10
14y = -7
divide both sides by 14
y = -½

substitute this value for y back into the first equation

5x + (4 x -½) = 3
5x – 2 = 3
add 2 to both sides
5x = 5
divide both sides by 5
x = 1

we have **x = 1 and y = -½**

to check substitute these values back into the second equation
1 – (2 x -½) = 1 - -1 = 1 + 1 = 2✓

substitution method

rearrange the second equation to make x the subject

$x = 2y + 2$

now substitute this value for x into the first equation

$(5 \times (2y + 2)) + 4y = 3$

$10y + 10 + 4y = 3$

$14y + 10 = 3$

subtract 10 from both sides

$14y = -7$

divide both sides by 14

$y = -½$

substitute this value for y back into the rearranged second equation

$x = 2y + 2$

$x = (2 \times -½) + 2 = -1 + 2$

$x = 1$

we have **x = 1 and y = -½**

to check substitute these values back into the first equation

$(5 \times 1) + (4 \times -½) = 5 - 2 = 3$✓

Question 6

Solve the following simultaneous equations

2x – 3y = 3
3x + 6y = 1

Answer:

We can solve linear simultaneous equations by elimination or by substitution

Elimination method

We need to have either the same number for x or the same number for y in both equations. We can do this by multiplying the first equation by 2 (to get 6y)
We must multiply every term by 2
4x – 6y = 6
3x + 6y = 1 unchanged
If we add these two equations then the y terms will disappear (be eliminated)
7x = 7
divide both sides by 7
x = 1
now put this back into the first equation to find y
(2 x 1) – 3y = 3
2 – 3y = 3
add 3y to both sides
2 = 3 + 3y
3 + 3y = 2
subtract 3 from both sides
3y = -1
divide both sides by 3
y = -1/3

x = 1 and y = -1/3
to check put both of these back into the second equation: (3 x 1) + (6 x $\frac{-1}{3}$) = 3 – 2 = 1✓

substitution method

rearrange one of the equations to make x or y the subject
here I have rearranged the first to make x the subject
$2x = 3 + 3y$, $x = ½(3 + 3y)$
now substitute this in to the second equation in place of x
$(3 \times (½(3 + 3y)) + 6y = 1$
$\frac{3}{2}(3 + 3y) + 6y = 1$
multiply by 2 (so we don't have fractions)
$3(3 + 3y) + 12y = 2$
expand the brackets
$9 + 9y + 12y = 2$
$9 + 21y = 2$
subtract 9 from both sides
$21y = -7$
divide both sides by 21
$y = \frac{-7}{21} = \frac{-1}{3}$
now put back into the first equation to find x
$2x - (3 \times \frac{-1}{3}) = 3$
expand brackets
$2x + 1 = 3$
subtract 1 from both sides
$2x = 2$
divide both sides by 2
$x = 1$

$x = 1$ and $y = -1/3$
to check put both of these back into the second equation: $(3 \times 1) + (6 \times \frac{-1}{3}) = 3 - 2 = 1$✓

Question 7

Solve the following simultaneous equations

$2x - 5y = 13$
$6x + 3y = 3$

Answer:

We can solve by elimination or by substitution. Elimination method would be better here as we can't easily express x or y in terms of the other.
Both methods are set out below.

Elimination
Label the equations eqn 1 and eqn 2
Multiply eqn 1 by 3 (so that we have the same number of xs in both equations)
$6x - 15y = 39$ eqn 3
$6x + 3y = 3$ eqn 2

Start with eqn 2 and subtract eqn 3 (done this way around so that end up with a positive number of ys)

$3y - -15y = 3 - 39$
$18y = -36$
divide both sides by 18
$y = -2$
now put back into eqn 1
$2x - (5 \times -2) = 13$
$2x + 10 = 13$
subtract 10 from both sides
$2x = 3$
divide both sides by 2
$x = 1.5$

$x = 1.5$, $y = -2$
Put both back into eqn 2 to see if it works:
$(6 \times 1.5) + (3 \times -2) = 9 - 6 = 3$✓

Substitution Method

Label the equations eqn 1 and eqn 2

Rearrange eqn 2 to make y the subject

$3y = 3 - 6x$

divide by 3

$y = 1 - 2x$

substitute this value for y into eqn 1

$2x - 5(1 - 2x) = 13$

expand the brackets

$2x - 5 + 10x = 13$

group terms

$12x - 5 = 13$

add 5 to both sides

$12x = 18$

divide both sides by 12

$x = 1.5$

now put back into the rearranged equation for y

$y = 1 - 2x$ to give $y = 1 - (2 \times 1.5) = 1 - 3 = -2$

$x = 1.5$, $y = -2$

Put both back into eqn 1 to see if it works:

$(2 \times 1.5) - (5 \times -2) = 3 - -10 = 13$✓

Question 8

Solve the following simultaneous equations

$y = x^2$
$y = 7x - 10$

Answer:

We have two equations and both are equal to y so we can put the two equations equal to each other (thus eliminating the y)

$x^2 = 7x - 10$
subtract $7x$ from both sides
$x^2 - 7x = -10$
add 10 to both sides
$x^2 - 7x + 10 = 0$
we have a quadratic to solve
we can factorise if we can find two numbers that multiply to give 10 and add to give -7
the two numbers are -2 and -5
$(x - 2)(x - 5) = 0$
If two numbers or brackets multiply to make 0 then either one of the brackets must equal 0
If $x - 2 = 0$ then $x = 2$
If $x = 5 = 0$ then $x = 5$

We have $y = x^2$
so when $x = 2$, $y = 4$ or when $x = 5$, $y = 25$
we need to check both these answers by putting them into the second equation
$y = (7 \times 2) - 10 = 14 - 10 = 4$✓ or $y = (7 \times 5) - 10 = 35 - 10 = 25$✓

$x = 2$, $y = 4$ or $x = 5$, $y = 25$

Question 9

Solve the following simultaneous equations

$y = 2x^2$
$y = 3x + 14$

Answer:

For both equations we have y as the subject so if they both equal y then they will equal each other

$2x^2 = 3x + 14$
rearrange to form a quadratic in x and with the quadratic equalling 0
subtract $3x$ from both sides
$2x^2 - 3x = 14$
subtract 14 from both sides
$2x^2 - 3x - 14 = 0$
Try to factorise
first multiply 2 by -14 to get -28, we must find two numbers that multiply to give -28 and combine to give -3
these two numbers are +4 and -7
rewrite the quadratic splitting the middle term into $+4x$ and $-7x$
$2x^2 + 4x - 7x - 14 = 0$
factorise in pairs
$2x(x + 2) - 7(x + 2) = 0$
we should have the same thing in both brackets which we do $(x + 2)$
now factorise again with $(x + 2)$ on the outside
$(x + 2)(2x - 7) = 0$
If two things multiply together to give 0 then one or the other of them must be 0
if $x + 2 = 0$ then $x = -2$
if $2x - 7 = 0$ then $2x = 7$ and $x = 3.5$
now put these x values back into either of the two original equations to get y
when $x = -2$, $y = 2 \times (-2)^2 = 2 \times 4 = 8$
when $x = 3.5$, $y = 2 \times 3.5^2 = 24.5$

To check, see if you get the same answer for y if put into the second equation
Y = (3 x -2) + 14 = -6 + 14 = 8✓, y = (3 x 3.5) + 14 = 10.5 + 14 = 24.5✓

$x = -2$, $y = 8$
or
$x = 3.5$, $y = 24.5$

Question 10

Solve the following simultaneous equations

y = 2*x*
4*x* – 5y = 9

Answer:

we can substitute the expression for y into the second equation
4*x* – (5 x 2*x*) = 9
4*x* – 10*x* = 9
-6*x* = 9
divide both sides by -6
x = - 1.5
put back into first equation y = 2*x*
y = 2 x -1.5 = -3

x = -1.5, y = 3

To check: put both values into the second equation and see if it works
(4 x -1.5) – (5 x -3) = -6 –15 = -6 + 15 = 9✓

Question 11

a) Solve the following simultaneous equations

$2x$ – 3y = 9
$5x$ + 4y = 1

b) write down the co-ordinates of the point of intersection of the two lines $2x$ – 3y = 9 and $5x$ + 4y = 1

Answer:

a)We can solve these by elimination or by substitution

By elimination
We want the same number of x or the same number of y
Multiply the top equation by 5 and the bottom equation by 2 then we will have $10x$ in both
$10x$ – 15y = 45
$10x$ + 8y = 22
we need to subtract the first equation from the second equation to eliminate the x terms
8y - - 15y = 22 – 45
8y + 15y = -23
23y = -23
divide both sides by 23
y = -1
now put this back into the first equation to get x
$2x$ – (3 x -1) = 9
$2x$ - -3 = 9
$2x$ + 3 = 9
subtract 3 from both sides
$2x$ = 6
divide both sides by 2
x = 3

We have x = 3 and y = -1

To check: put back into the second equation
(5 x 3) + (4 x -1) = 15 + -4 = 15 – 4 = 11✓

By substitution

rearrange the first equation to make y the subject

add 3y to both sides

$2x = 3y + 9$

rewrite

$3y + 9 = 2x$

subtract 9 from both sides

$3y = 2x - 9$

divide both sides by 3

$y = \frac{2x-9}{3}$

substitute this value for y into the second equation

$5x + (4 \times \frac{2x-9}{3}) = 11$

multiply by 3

$15x + 4(2x - 9) = 33$

expand

$15x + 8x - 36 = 33$

group terms

$23x - 36 = 33$

add 36 to both sides

$23x = 69$

divide both sides by 23

$x = 3$

Now substitute this value for x back into $y = \frac{2x-9}{3}$

$y = \frac{(2 \times 3)-9}{3} = \frac{6-9}{3} = \frac{-3}{3} = -1$

We have $x = 3$ and $y = -1$

To check: put back into the second equation

$(5 \times 3) + (4 \times -1) = 15 + -4 = 15 - 4 = 11$ ✓

b) this is what we have just done

(3, -1)

Question 12

Solve the following simultaneous equations

6x + 2y = -3
4x – 3y = 11

Answer:

We can solve simultaneous equations by elimination or by substitution
Elimination method
Label each equation eqn 1 and eqn 2
We need to have the same number for x or the same number for y
Multiply eqn 1 by 2 and multiply eqn 2 by 3 (that way we will have 12x in both equations)

12x + 4y = -6 eqn 3
12x – 9y = 33 eqn 4
subtract eqn 4 from eqn 3

4y - - 9y = -6 – 33
13y = -39
divide both sides by 13
y = -3
now put this value for y back into eqn 1
6x + (2 x -3) = -3
6x – 6 = -3
add 6 to both sides
6x = 3
divide both sides by 6
x = 0.5

x = 0.5 and y = -3
Check by substituting both values into eqn 2: (4 x 0.5) – (3 x -3) = 2 - - 9 = 2 + 9 = 11✓

Substitution method

Label each equation eqn 1 and eqn 2

rearrange eqn 1 to make y the subject

$2y = -3 - 6x$

divide by 2

$y = -1.5 - 3x$ eqn 3

now substitute eqn 3 into eqn 2

$4x - 3(-1.5 - 3x) = 11$

$4x + 4.5 + 9x = 11$

$13x + 4.5 = 11$

subtract 4.5 from both sides

$13x = 6.5$

divide both sides by 13

$x = 0.5$

now substitute this value for x back into eqn 3

$y = -1.5 - (3 \times 0.5) = -1.5 - 1.5 = -3$

$x = 0.5$ and $y = -3$

Check by substituting both values into eqn 2: $(4 \times 0.5) - (3 \times -3) = 2 - -9 = 2 + 9 = 11$✓

Question 13

Solve the following simultaneous equations

3x + 2y = 8
2x + 5y = -2

Answer:

We can solve by elimination or by substitution

By elimination

We need the same number for x or the same number for y. We can do this by multiplying the first equation by 2 and the second equation by 3. This will give 6x in both cases. (We could have multiplied the first by 5 and the second by 2 to give 10y).

3x + 2y = 8 x 2 gives 6x + 4y = 16 equation 3
2x + 5y = -2 x 3 gives 6x + 15y = -6 equation 4

Now we have the same number for x in both equations
Subtract equation 3 from equation 4 to give

15y – 4y = -6 – 16
11y = -22
divide both sides by 11
y = -2

put this value for y back into the first equation to give
3x + 2(-2) = 8
3x – 4 = 8
add 4 to both sides
3x = 12
divide both sides by 3
x = 4

now put both values back into the second equation to make sure it works
2(4) + 5(-2) = -2
8 – 10 = -2✓

x = 4, y = -2

By substitution

First rearrange one of the equations so that either x or y is the subject

$3x = 8 - 2y$

$x = \frac{8-2y}{3}$

Substitute this value for x into the other equation

$(2 \times \frac{8-2y}{3}) + 5y = -2$

multiply throughout by 3 to get rid of the fractions

$2 \times (8 - 2y) + 15y = -6$

expand the brackets

$16 - 4y + 15y = -6$

$16 + 11y = -6$

subtract 16 from both sides

$11y = -22$

divide both sides by 11

$y = -2$

put this value for y back into the first equation to give

$3x + 2(-2) = 8$

$3x - 4 = 8$

add 4 to both sides

$3x = 12$

divide both sides by 3

$x = 4$

now put both values back into the second equation to make sure it works

$2(4) + 5(-2) = -2$

$8 - 10 = -2$ ✓

$x = 4$, $y = -2$

Question 14

Solve the following simultaneous equations

$4x$ + y = -1
$4x$ - 3y = 7

Answer:

To solve simultaneous equations you can either solve by elimination or by substitution:

Elimination
We need the same number of x or the same number of y. We have 4x in both equations. If we subtract one equation from the other then the x terms will have been eliminated.

($4x$ + y = -1)
- ($4x$ – 3y = 7)
y - -3y = -1 – 7

4y = -8
now divide both sides by 4
y = -2
substitute this value for y back into the first or the second equation. I am putting it back into the first equation.

$4x$ + -2 = -1
$4x$ – 2 = -1
add 2 to both sides of the equation
$4x$ = 1
divide both sides of the equation by 4
$x = \frac{1}{4}$

we have x = $\frac{1}{4}$ and y = -2

substitute both of these back into the other equation to make sure it works (I need to put back into the second equation)
(4 x $\frac{1}{4}$) – (3 x -2) = 1 - -6 = 1 + 6 = 7✓

Substitution

First we need to rearrange one of the equations so that either x or y is the subject. The easiest one to rearrange here is the first equation.

$4x + y = -1$

subtract $4x$ from both sides

$y = -1 - 4x$

substitute this value for y into the second equation

$4x - (3(-1 - 4x) = 7$

$4x - (-3 - 12x) = 7$

$4x + 3 + 12x = 7$

$16x + 3 = 7$

subtract 3 from both sides

$16x = 4$

divide both sides by 16

$x = \frac{4}{16} = \frac{1}{4}$

going back to the rearranged equation $y = -1 - 4x$ and substituting in our value for x we get

$y = -1 - (4 \times \frac{1}{4}) = -1 - 1 = -2$

we have $x = \frac{1}{4}$ and $y = -2$

substitute both of these back into the other equation to make sure it works (I need to put back into the second equation)

$(4 \times \frac{1}{4}) - (3 \times -2) = 1 - -6 = 1 + 6 = 7$✓

Question 15

Solve the following simultaneous equations

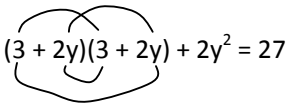
$x = 3 + 2y$
$x^2 + 2y^2 = 27$

Answer:

We have to use the substitution method. The elimination method won't work as one equation has x and y, the other has x^2 and y^2

substitute the equation for x into the second equation

$(3 + 2y)^2 + 2y^2 = 27$
expand



$9 + 4y^2 + 6y + 6y + 2y^2 = 27$
group terms
$6y^2 + 12y + 9 = 27$
subtract 27 from both sides
$6y^2 + 12y - 18 = 0$
divide by 6
$y^2 + 2y - 3 = 0$
factorise
$(y - 1)(y + 3) = 0$
if two things multiply to give 0 then either one of them must equal 0
If $y - 1 = 0$, then $y = 1$
If $y + 3 = 0$, then $y = -3$

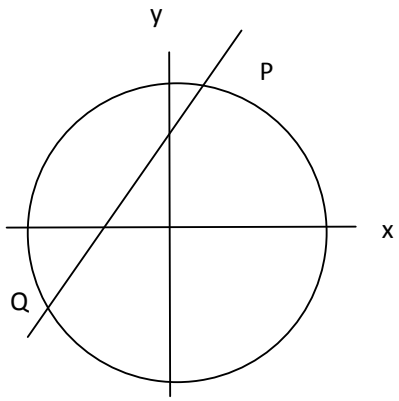
now we have y, we can substitute this value back into the first equation to get x
when $y = 1$
$x = 3 + (2 \times 1) = 3 + 2 = 5$
when $y = -3$
$x = 3 + (2 \times -3) = 3 + -6 = 3 - 6 = -3$

$x = 5$, $y = 1$
or
$x = -3$, $y = -3$

to check put these values into the second equation:
$5^2 + (2 \times 1^2) = 25 + 2 = 27$✓
$(-3)^2 + (2 \times (-3)^2) = 9 + (2 \times 9) = 9 + 18 = 27$✓

Question 16

The circle $x^2 + y^2 = 16$ and the line $y = x + 2$ meet at the points P and Q.



a)Show algebraically that the x co-ordinates of points P and Q satisfy the equation $x^2 + 2x - 6 = 0$

b) write the equation $x^2 + 2x - 6 = 0$ in the form $(x + a)^2 - b = 0$

c) Hence, or otherwise, solve the equation $x^2 + 2x - 6 = 0$ and give your answers in surd form.

Answer:

a)We must use substitution to solve these simultaneous equations:
replace y in the first equation with x + 2

$x^2 + (x + 2)^2 = 16$
$x^2 + (x + 2)(x + 2) = 16$
expand the brackets
$x^2 + (x + 2)(x + 2) = 16$
$x^2 + x^2 + 2x + 2x + 4 = 16$

group terms
$2x^2 + 4x + 4 = 16$
subtract 16 from both sides
$2x^2 + 4x - 12 = 0$
divide both sides by 2
$x^2 + 2x - 6 = 0$

b) $(x + 1)^2 - (1)^2 - 6 = 0$
grouping terms
$(x + 1)^2 - 7 = 0$

c) add 7 to both sides
$(x + 1)^2 = 7$
square root both sides
$x + 1 = \pm\sqrt{7}$
subtract 1 from both sides

$x = -1 \pm\sqrt{7}$

Question 17

a) show that the x co-ordinates of the solutions of the simultaneous equations

$2x + y = 3$ and $x^2 + y^2 = 5$

satisfy the equation $5x^2 - 12x + 4 = 0$

b) solve the equation $5x^2 - 12x + 4 = 0$

Answer:

a)We have to solve this by substitution. If we rearrange $2x + y = 3$ to make y the subject

$2x + y = 3$

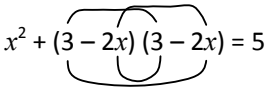
subtract 2x from both sides

$y = 3 - 2x$

now substitute this value for y into $x^2 + y^2 = 5$

$x^2 + (3 - 2x)^2 = 5$

$x^2 + (3 - 2x)(3 - 2x) = 5$



expand the brackets

$x^2 + 9 + 4x^2 - 6x - 6x = 5$

group terms

$5x^2 - 12x + 9 = 5$

subtract 5 from both sides

$5x^2 - 12x + 4 = 0$

b) first try to factorise:
this is a harder quadratic as there is a number in front of the x^2 so we must first multiply the 5 by the 4 to get 20. Then we need to find two numbers that multiply to give +20 and combine to give -12. These two numbers are -2 and -10.
Rewrite the equation splitting the $-12x$ term into $-2x$ and $-10x$
$5x^2 - 2x - 10x + 4 = 0$
factorise in pairs
$x(5x - 2) - 2(5x - 2) = 0$
we should have the same factor for both pairs (which we do and it is $(5x - 2)$)
as $(5x - 2)$ goes into both terms we can factorise again
$(5x - 2)(x - 2) = 0$
If you multiply two things together to get 0 then one of them must equal 0
So $5x - 2 = 0$ or $x - 2 = 0$

$5x - 2 = 0$
add 2 to both sides
$5x = 2$
divide both sides by 5
$x = \frac{2}{5}$

$x - 2 = 0$
add 2 to both sides
$x = 0$

We have $x = 2$ or $x = \frac{2}{5} = 0.4$

Alternatively, we could have used the quadratic formula here (given in formulae sheet)
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where a = 5, b = -12 and c = 4

$$x = \frac{12 \pm \sqrt{144 - (4\text{x}5\text{x}4)}}{2\text{x}\ 5}$$

$$x = \frac{12 \pm \sqrt{144 - 80}}{10}$$

$$x = \frac{12 \pm \sqrt{64}}{10}$$

$$x = \frac{12 \pm 8}{10}$$

So $x = 2$ or 0.4

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